Indian Statistical Institute, Bangalore
B. Math (Hons.) Third Year
Second Semester - Complex Analysis

Back-paper Exam — Date: 03rd June 2025
Maximum marks: 100 — Duration: 3 hours

Each question carries 10 marks

1. If $f, g \in H(\Omega)$, prove that $f \pm g$ and $fg \in H(\Omega)$.
2. (a) Determine all $f \in H(\mathbb{C})$ such that $|f(z)| = |z|$ for all $z \in \mathbb{C}$ *(Marks: 5)*.
   (b) Determine all entier functions f such that $f(z) + \overline{f(z)}$ is constant.
3. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ have radius of convergence R. Then prove that f is holomorphic and $f'(z) = \sum_{n=1}^{\infty} n a_n z^{n-1}$ on $|z| < R$.
4. Prove that $f \in H(\Omega)$ is representable by a power series on Ω.
5. If $f \in H(\Omega)$ and $\gamma \subset \Omega$ is the boundary of a circle in Ω, find all $a \in \Omega \setminus \gamma^*$ so that $f^{(n)}(a) = \frac{n!}{2\pi i} \int_\gamma \frac{f(z)}{(z-a)^{n+1}} dz$. Justify your answer.
6. State and prove Schwarz lemma.
7. If f has a simple pole at $z = 0$, find $\text{res}(fg; 0) - g(0)\text{res}(f; 0)$ for any analytic function g in the unit disc. Jusify your answer.
8. Find the number of zeros of $p(z) = 6z^6 - 3z^3 + 2z$ and $q(z) = 2z^8 - 6z^7 + z - 3$ inside the unit circle. Justify your answer.
9. Evaluate $\int_0^\pi \frac{d\theta}{\cos\ \theta + \sqrt{2}}$ using residue theorem.
10. If $f \in H(\Omega)$, find residues of $\frac{f'}{f}$ at its poles in Ω. Justify your answer.